UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2007 question paper

4037 ADDITIONAL MATHEMATICS

4037/01 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW -1,2 This is deducted from A or B marks when essential working is omitted.

PA -1 This is deducted from A or B marks in the case of premature approximation.

S -1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX -1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1 $\mathbf{A}=\begin{pmatrix}2 & -1\\ 3 & 1\end{pmatrix}$, $\mathbf{A}^2=\begin{pmatrix}1 & -3\\ 9 & -2\end{pmatrix}$ $\begin{pmatrix}1 & -3\\ 9 & -2\end{pmatrix} + m\begin{pmatrix}2 & -1\\ 3 & 1\end{pmatrix} = n\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ $1 + 2m = n$ and $-3-m=0$ $\rightarrow$ $m = -3$ and $n = -5$	M1 A1 M1 A1 [4]	Reasonable attempt (needs 2 correct) All correct Identity matrix must be correct Equating their elements once co.
2 $\left(\frac{1}{1-\cos\theta}-\frac{1}{1+\cos\theta}\right)\equiv 2\operatorname{cosec}\theta\cot\theta$ Manipulation of fractions $(1-c)(1+c) = s^2$ used $\frac{2\cos\theta}{\sin^2\theta} \Rightarrow 2\operatorname{cosec}\theta\cot\theta$ All correct	 M1 B1 M1 A1 [4]	 (1−c)(1+c) in denominator +reasonable attempt at numerator (ignore signs) Knowledge of cot and cosec When all correct a.g Beware fortuitous answers.
3 (i) $p=\frac{\sqrt{3}+1}{\sqrt{3}-1} \rightarrow p=\frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$ $\rightarrow$ $p=\frac{3+2\sqrt{3}+1}{3-1}$ = 2 + √3 **(ii)** either $p-\frac{1}{p}$ = 2 + √3 − $\frac{1}{2+\sqrt{3}}$ or $p-\frac{1}{p}=\frac{p^2-1}{p}$ $\rightarrow$ 2√3	M1 A1 A1 [3] M1 A1 [2]	× top and bottom by √3 + 1 Denominator = 2 co Complete method. co.
4 (i) 4 men from 9 = ${}_9C_4$ (126) 4 women from 6 = ${}_6C_4$ (15) Multiply together $\rightarrow$ 1890 **(ii)** One twin included (${}_7C_3\times{}_6C_4$) To include other twin ×2 $\rightarrow$ 1050	 B1 M1 A1 [3] M1 DM1 A1 [3]	 For either ${}_9C_4$ or ${}_6C_4$ Product of 2 ${}_nC_r$s. co For 2 ${}_nC_r$s. Two times his first answer. co
5 (i) Resultant vel = (960**i** +400**j**) ÷ 4 $\rightarrow$ (240**i** +100**j**) **v** (still air) = (240**i** +100**j**) − **wind** = 300**i** + 40**j** **(ii)** tanθ = 40 ÷ 300 ($\rightarrow$ 7.6º) $\rightarrow$ Bearing of 082º (awrt 82º)	M1 A1 M1 A1 [4] M1 A1 [2]	Division of distance by time co (could be wind × 4) then ÷ 4 later Needs subtraction co Use of tan with their 2 components Not 960**i** +400**j**

6 (i) $\frac{dy}{dx} = \frac{6}{\sqrt{4x+1}}$ $y = \frac{6(4x+1)^{\frac{1}{2}}}{\frac{1}{2}} \div 4$ (+c) Uses (6,20) → $c = 5$ $(y = 3\sqrt{4x+1} + 5)$	B1 B1 M1 A1 [4]	For an expression involving $(4x+1)^{\frac{1}{2}}$ For all correct Uses (6,20) in an integration involving $(4x+1)^{\frac{1}{k}}, k \neq -\frac{1}{2}$ co (do not mark after +5)
(ii) Perp to −½ = 2	M1	Use of $m_1m_2=-1$, with attempt to solve co on y-value, using $x = 2$
$\frac{6}{\sqrt{4x+1}} = 2 \rightarrow x = 2, y = 14$	A1√	
Eqn → $y - 14 = -½(x - 2)$ or $2y+x=30$ → (0, 15) and (30, 0)	M1 A1 [4]	Correct method for line co
7 (i) $2^{2x} = 2^{x+2} + 5$ $2^{2x} = u^2$ $2^{x+2} = 4u$ Solution of quadratic $u^2=4u+5$ $2^x = 5 \rightarrow x = \lg5 \div \lg2$ → $x = 2.32$	B1 B1 M1 M1 A1 [5]	co co Correct method of solution of quad=0 From $2^x = k$ to x by correct method co – loses if more than one answer given.
(ii) $2\log_9 3 + \log_5(7y - 3) = \log_2 8$. $2\times½ + \ldots = 3$ $\log_5(7y - 3) = 2$ $(7y - 3) = 25 \rightarrow y = 4$	B1 B1 M1 A1 [4]	For ½ For RHS = 3 From $\log_5$ to $5^p = k$. co
8 **(a)** f(1) = 1−11+k-30 → $k - 40$ f(2) = 8−44+2k-30 → $2k - 66$ f(1) = 4f(2) → $k = 32$	M1 A1 M1 A1 [4]	Uses either $x = 1$ or 2, not −1 or −2. Both correct, unsimplified. Linked + solution – allow if 4 on LHS co
(b) $x^3 - 4x^2 - 8x + 8 = 0$ Tries for a first solution → $x = -2$ Divides by (x − his first solution) → $x^2 - 6x + 4 = 0$ → $x = \frac{6 \pm \sqrt{20}}{2}$ → $3 \pm \sqrt{5}$	M1 A1 M1 DM1 A1 [5]	Search shown for M,$x = -2$ gets M1A1. Correct method. Correct method for soln of quadratic Must be simplified.

9 (i)

x	2	4	6	8	10
y	14.4	10.8	11.2	12.6	14.4
xy	29	43	67	101	144
x^2	4	16	36	64	100

M1 Knows what to do.
A1 Mark from graph – 5 points are in line. [2]

(ii) Gradient 1.2 (±0.1)
'y' intercept (± 2)

B1 co
B1 co

$\rightarrow y = 1.2x + \frac{24}{x}$

M1 $xy =$ (their grad)x + (their intercept)
A1 [4]

(iii) From graph $xy = 83 \rightarrow x^2 = 49$
Valid method to obtain y
$y = 11.6 – 12.2$

M1 Reads on vertical axis at 83
M1 Valid method to obtain y
A1 co [3]

10 (i) $BC = 2(10\sin 0.4) = 7.79$

M1 A1 Any correct method – cos rule ok. [2]

(ii) $\angle ABC = \frac{1}{2}(\pi - 0.8) = 1.17$ rads
Arc $CD = 7.79 \times 1.17$, Arc $BC = 10 \times 0.8$
$\rightarrow$ P = sum of the arcs + BD (=7.79)
$\rightarrow$ $P = 24.9$

B1 Anywhere in the question.
M1 Use of $s = r\theta$ in either arc.
M1 Overall plan – arc CD + arc BC + BD
A1 co. [4]

(iii) Area sector $BDC = \frac{1}{2}(7.79)^2 \times 1.17$
Area segment on $BC = \frac{1}{2}.10^2(0.8 - \sin 0.8)$

$\rightarrow$ Shaded area = 39.6 or 39.7

M1 Use of $A = \frac{1}{2}r^2\theta$ for sector BDC
B1 B1 for $0.5(10)^2 0.8$
B1 B1 for $0.5(10)^2 \sin 0.8$
A1 co [4]

11 EITHER

(i) $y = xe^{2x}$ $\quad d/dx(e^{2x}) = 2e^{2x}$
$\rightarrow$ $dy/dx = e^{2x} + 2x\,e^{2x}$

B1 Anywhere – even if product not used
M1A1 Use of correct formula for "uv". co

$\rightarrow d^2y/dx^2 = 2e^{2x} + 2e^{2x} + 4xe^{2x}$

M1A1 Use of product formula again. co. [5]

(ii) $dy/dx = 0$ when $1 + 2x = 0 \rightarrow x = -\frac{1}{2}$

$\rightarrow$ $y = -\frac{1}{2}e^{-1} = -\frac{1}{2e}$.

M1 A1 Sets his dy/dx to 0 and tries to solve.
A1 co – ag – beware fortuitous results. [3]

(iii) If $x = -\frac{1}{2}$ $\rightarrow$ +ve result
$\rightarrow$ Minimum
(or gradient goes −,0,+)

(or y value to left or right of $(-\frac{1}{2}) > -\frac{1}{2e}$)

M1 Looks at sign.
A1 Correct deduction from correct x.
(or by any other valid method) [2]

11 OR		
(i) d/dx(ln$x$) = 1/x $\frac{d}{dx}\left(\frac{\ln x}{x^2}\right) = \frac{x - 2x\ln x}{x^4} = \frac{1-2\ln x}{x^3}$	B1 M1 A1 [3]	Anywhere – even if quotient not used Use of correct quotient formula co
(ii) dy/dx = 0 → ln$x$ = ½ → x = √e → y = ln(√e)÷e = $\frac{1}{2e}$.	M1 A1 A1 [3]	Sets his dy/dx to 0 and tries to solve. co – ag – beware fortuitous results.
(iii) $\frac{\ln x}{x^2} = \int\left(\frac{1}{x^3}\right)dx - \int\frac{2\ln x}{x^3}dx$	M1	Recognition that integration is the reverse of differentiation.
$\int\frac{\ln x}{x^3}dx$ = ½ × [$\int\left(\frac{1}{x^3}\right)dx - \frac{\ln x}{x^2}$] → = $\frac{1}{2}\left(\frac{x^{-2}}{-2} - \frac{\ln x}{x^2}\right)$ + c	B1 B1 A1 [4]	B1 for ½. B1 for (x^{-2}) ÷ (−2) All ok including +c.
DM1 for quadratic equation. Equation must be set to 0 if using formula or factors. Formula. Must be correct – ignore arithmetic and algebraic slips.		Factors Must attempt to put quadratic into 2 factors. Each factor then equated to 0.